



### Optimizing the Slope and Azimuth Angles of Solar Collectors and investigate effect of earth reflection coefficient on its in Mazandaran-Iran

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### **Abstract**

The performance of a solar collector is affected by its orientation, its slope angle with the horizontal plane and the earth reflection coefficient. In the present article, the optimum slope angle and the optimum slope and azimuth angles of solar collectors are calculated utilizing Liu, Klein and Hay models in Mazandaran-Iran. The results showed that the energy reached to the collector oriented at optimum slope angle increased about 25.4% comparing to the horizontal one utilizing Liu model. This was about 31.28% and 57.76% for the collector oriented at optimum slope and azimuth angles utilizing Klein and Hay models, respectively. Moreover, the results showed that the collector oriented at Hay optimum angles received significantly more energy. Finally, the effect of earth reflection coefficient on the maximum radiation reached by the collector oriented at Klein and Hay optimum angles was investigated. The results showed that increasing of the earth reflection coefficient increases the maximum reached radiation.

Keywords: Solar energy, Solar collector, Slope angle, Azimuth angle, Optimum angles, earth reflection coefficient

### Introduction

Solar systems, like any other systems, need to be operated with the maximum possible performance. This can be achieved by precise design, construction, installation, and orientation. The orientation of the collector is described by its slope and azimuth angles. Generally, in the northern hemisphere, the installed collectors are oriented to the south. Many researchers have been carried out to calculate the best slope angle for such systems. Some of these are, for example,  $\beta_{opt} = \phi + 20^{\circ}$  (HC Hottel,1954),  $\beta_{opt} = \phi + (10 \rightarrow 30^{\circ})$  (Course Symp,1954),  $\beta_{opt} = \phi + 10^{\circ}$  (GOG Lo"f, RA Taybout,1973) and  $\beta_{opt} = \phi - 10^{\circ}$  (J. Kern J, I. Harris,1975). Many investigations show that the optimum tilt angle is related to the site latitude. For instance, Duffie et al. suggested that  $\beta_{opt} = (\phi + 15) \pm 15$  ( J. A. Duffie, W. A. Beckman,1982). Mujahid obtained the optimum angle for different latitudes from  $10^{\circ}$  to  $50^{\circ}$  and showed that only 10% of energy is approximately wasted by fixing the solar collector at the optimum slope angle (M. Mujahid,1994). Ibrahim showed that changing the slope angle monthly causes it to receive more energy compared to a fixed solar collector during the year (D. Ibrahim, 1995). Bari obtained the optimum slope angles of different places in Malesia (S. Bari, 2000). Yakup et al. calculated the monthly optimum slope angle and also obtained the hourly, daily, seasonal, and yearly optimum slope angles in Dar-Salam (M. Yakup, A. Q. Malik, 2001). Sharia et al. calculated the optimum slope angle using the TRNSYS software (University of Wisconsin, Madison, WI) in Ordun (Shariah, Al-Akhras M-Ali, I. A. Omari,2002). Qiu et al. analyzed the optimum angle for some cities of Macrocosm and reported the optimum slope angle  $\beta_{ont} = \phi \pm 10$  (G. Qiu, S. B. Riffat,2003). Alonge et al. represented a mathematical model in some cities in Nigeria (F. Alonge, K. Oje, K,2006). Ulgen calculated the optimum angle in the Izmir of Turkia (K. Ulgen,2006). Elminir et al. did the same in Egypt (Elminir et al, 2006). Gopinathan et al. represented the optimum slope and the azimuth angle by a new mathematical method for South Africa (K. K. Gopinathan, N. B. Maliehe, M. I. Mpholo, 2007). Samareh Salavati Pour et al. developed a computer program and calculated the monthly optimum tilt angle and the monthly optimum slope and azimuth angles of solar collectors during the year in Isfahan-Iran (Samareh Salavati Pour et al.,2010&2011).

In the present paper, in order to obtain the maximum monthly total solar energy during a year, the optimum tilt angle alone and the optimum slope with azimuth angles consideration have been calculated utilizing LIU, K-T and Hay





models in Mazandaran-Iran. Then, the total solar energy for these three cases is compared. Finally, the effect of earth reflection coefficient on the maximum radiation received by the collector is investigated.

### Mathematical model

The information of solar radiation in different places exists as the radiation on the flat surface. Using the following mathematical models, solar radiation on sloped surface is calculated. Total daily radiation on a sloped surface  $H_T$  is the sum of beam radiation  $H_B$ , diffuse radiation  $H_S$  and reflective radiation  $H_R$ .

$$H_T = H_B + H_S + H_R \tag{1}$$

The first method for calculation  $H_T$  in this paper is an isotropic method which is introduced by Liu (B. Y. H. Liu, R. C. Jordan, 1991). The monthly average radiation on tilt surface is calculated using this method. This approach is merely used for a collector with zero azimuth angles. In this method beam radiation  $H_B$ , diffuse radiation  $H_S$  and reflective radiation  $H_R$  are calculated as below:

$$H_{B} = H(1 - \frac{H_{d}}{H})R_{b} \tag{2-1}$$

$$H_S = H_d(\frac{1 + \cos \beta}{2}) \tag{2-2}$$

$$H_R = H\rho_g(\frac{1-\cos\beta}{2})\tag{2-3}$$

So total daily radiation on tilt surface is:

$$H_T = H(1 - \frac{H_d}{H}) + H_d(\frac{1 + \cos \beta}{2}) + H\rho_g(\frac{1 - \cos \beta}{2})$$
(3)

Where  $\beta$  is tilt angle,  $\rho_g$  is earth reflective coefficient,  $H_T$  is total radiation on tilt surface and H is total radiation on flat surface. In the recent equation  $R_b$  is the average beam radiation on sloped surface divided by average beam radiation on flat surface. For a surface with zero azimuth angle  $R_b$  is calculated as below:

$$R_{b} = \frac{\cos(\phi \pm \beta)\cos\delta\sin\overline{\omega}_{s} + (\pi/180)\overline{\omega}_{s}\sin(\phi \pm \beta)\sin\delta}{\cos(\phi)\cos\overline{\delta}\sin\omega_{s} + (\pi/180)\omega_{s}\sin(\phi)\sin\delta}$$

$$(4)$$

Sunset hour angle  $(\omega_s)$ , sunset hour angle for a sloped surface  $(\omega_s')$  and sun declination angle  $(\delta)$  are defined as below:

$$\omega_{\rm S} = \cos^{-1}(-\tan\phi\tan\delta) \tag{5}$$

$$\omega_{S}' = \min \begin{bmatrix} \cos^{-1}(-\tan\phi\tan\delta) \\ \cos^{-1}(-\tan(\phi\pm\beta)\tan\delta) \end{bmatrix}$$
 (6)

$$\delta = 23.45\sin(360\frac{284 + n}{365})\tag{7}$$

 $H_d$  in equation (4) is the diffuse radiation on flat surface and is calculated using bellow equation:

$$H_{d} = \begin{cases} H \times (1.391 - 3.56K_{T} + 4.189K_{T}^{2} - 2.137K_{T}^{3}) & if\omega_{S} < 1.42 \\ H \times (1.311 - 3.022K_{T} + 3.427K_{T}^{2} - 1.821K_{T}^{3}) & f\omega_{S} \ge 1.42 \end{cases}$$

$$(8)$$

 $K_T$  is the average radiation on flat surface to the average irradiance radiation and defined as below:

$$K_T = \frac{H}{H_o} \tag{9}$$

 $H_o$  Irradiance radiation is:





$$H_o = \frac{24 \times 3600 \times G_{SC}}{\pi} (1 + 0.033 \cos \frac{360n}{365}) \times (\cos \phi \cos \delta \sin \omega_s + \frac{\pi \omega_s}{180} \sin \phi \sin \delta)$$
 (10)

 $G_{\it SC}$  , sun constant radiation is about 1367(  $\it Mj/m^2$  ) [5].

All mentioned equations are used for both north and south hemisphere.

Hay presented another method to calculate total radiation with the effects of azimuth angle (J. E. Hay, 1979). In this method,  $R_b$  is the same for both north and south hemisphere, just the obtained data for January in north hemisphere are used for July in south hemisphere.

$$R_b = \{\cos\beta\sin\delta\sin\phi(\omega_{ss} - \omega_{sr})(\pi/180) - \sin\delta\cos\phi\sin\beta\cos\gamma(\omega_{ss} - \omega_{sr})(\pi/180) + \cos\phi\cos\delta\cos\beta(\sin\omega_{ss} - \sin\omega_{sr}) + \cos\delta\cos\gamma\sin\phi\sin\beta(\sin\omega_{ss} - \sin\omega_{sr}) - \sin\omega_{sr}\}$$
(22)

 $\cos \delta \sin \beta \sin \gamma (\cos \omega_{ss} - \cos \omega_{sr}) \} / \{ 2[\cos \phi \cos \delta \sin \omega_{s} + (\pi/180)\omega_{s} \sin \phi \sin \delta] \}$ 

$$if \gamma < 0 \begin{cases} \omega_{sr} = -\min \left[ \omega_{s}, \cos^{-1} \frac{AB + \sqrt{A^{2} - B^{2} + 1}}{A^{2} + 1} \right] \\ \omega_{ss} = \min \left[ \omega_{s}, \cos^{-1} \frac{AB - \sqrt{A^{2} - B^{2} + 1}}{A^{2} + 1} \right] \end{cases}$$
(23)

$$if \gamma > 0 \begin{cases} \omega_{sr} = -\min \left[ \omega_{s}, \cos^{-1} \frac{AB - \sqrt{A^{2} - B^{2} + 1}}{A^{2} + 1} \right] \\ \omega_{ss} = \min \left[ \omega_{s}, \cos^{-1} \frac{AB + \sqrt{A^{2} - B^{2} + 1}}{A^{2} + 1} \right] \end{cases}$$
(24)

$$A = \frac{\cos\phi}{\sin\gamma\tan\beta} + \frac{\sin\phi}{\tan\gamma} \tag{25}$$

$$B = \tan \delta \left( \frac{\cos \phi}{\tan \gamma} - \frac{\sin \phi}{\sin \gamma \tan \beta} \right) \tag{20}$$

Monthly total radiation due to this method is calculated as:

$$H_{T} = (H - H_{d})R_{b} + H\rho_{g}(\frac{1 - \cos\beta}{2}) + H_{d}\left\{\frac{H - H_{d}}{H_{o}}R_{b} + (\frac{1 + \cos\beta}{2})(1 - \frac{H - H_{o}}{H_{o}})\right\}$$
(26)

In this equation,  $H_o$  is irradiance radiation.

### **Results and discussion**

The needed parameters in the three explained models were provided from the Iranian Metrological Organization (IMO)-Mazandaran. Mazandaran is located in the northern latitude of  $35^{\circ}$  eastern longitude of  $52^{\circ}$  and height of 54m from the sea area. Solar radiation on a flat surface in the three explained models was obtained from the IMO for a period of seven years (2006–2013) in Mazandaran. A pyranometer shown in Figure 1. was used to measure the solar radiation on the flat surface. The total solar radiation on the horizontal surface is given in Table 1. As this tables shows, the maximum solar energy is in June, which is about 25.12 (w/m²), and the minimum is in December, which is about 7.78 (w/m²). The earth reflection coefficients are also given in Table 1. These coefficients for different ground covers are given in Table 2. In the present paper, these three models (Liu, KT and Hay) are used to calculate solar radiation on a sloped surface.







Figure 1- Pyranometer for measuring radiation energy

**Table 1-** Information of months

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Month	N [5]	$ ho_{g}$	$H(MJ/m^2)$			
Jan	17	0.27	9.47			
Feb	47	0.27	14.23			
March	75	0.27	15.77			
April	105	0.18	17.31			
May	135	0.18	22.68			
June	162	0.18	25.12			
July	198	0.18	23.51			
Aug	228	0.18	23.14			
Sep	258	0.18	20.58			
Oct	288	0.18	15.62			
Nov	318	0.18	9.51			
Dec	344	0.18	7.78			

Table 2- Earth reflective coefficient for different ground covers

<b>Ground Cover</b>	Reflectivity
Dry bare ground	0.3
Dry grass land	0.3
Desert sand	0.4
Snow	0.5-0.8

### 3.1 Monthly optimum slope angle

In the first part, using solar radiation on flat surface and equations, monthly average total radiation on the sloped surface is calculated. The result shows that monthly optimum slope angle for June and July is negative and collector must be set to south in these months.

### 3.2 Monthly optimum slope angle and azimuth angle

In the second part, the results of Klein and Hay models are used. The results show that the maximum energy due to KT model is in  $\beta$ =60° and  $\gamma$ =0° for January and in  $\beta$ =31° and  $\gamma$ =78° for July. Moreover, the maximum energy is in  $\beta$ =44° and  $\gamma$ =44° for March and is in  $\beta$ =38° and  $\gamma$ =44° for September. The results of the optimum tilt and azimuth the angles for all months due to KT and Hay models are illustrated in Table 4 and 5.





**Table 4-** Monthly average optimum slope and azimuth angle and maximum radiation on the collector in Mazandaran (KT Model)

(K1-Model)						
Month	$\beta^{\circ}$	$\gamma^{\circ}$	$H(MJ/m^2)$	Percent		
				of Gain		
				(%)		
Jan	60	0	16.72	76.52		
Feb	52	0	21.32	49.80		
March	42	42	19.48	23.54		
April	32	60	19.07	10.19		
May	31	74	23.77	4.83		
June	31	82	25.80	2.71		
July	31	78	24.36	3.61		
Aug	32	64	25.05	8.27		
Sep	38	44	24.55	19.28		
Oct	48	33	21.57	38.06		
Nov	55	0	15.32	61.06		
Dec	60	0	13.81	77.51		

**Table 5-** Monthly average optimum slope and azimuth angle and maximum radiation on the collector in Mazandaran (Hay-Model)

Month	$oldsymbol{eta}^\circ$	$\gamma^{\circ}$	$H(MJ/m^2)$	Percent of Gain (%)
Jan	58	0	19.79	108.97
Feb	51	0	25.21	77.14
March	35	0	23.48	48.90
April	35	1	22.90	32.27
May	35	1	26.31	16.00
June	35	180	28.53	13.56
July	35	180	26.48	12.64
Aug	35	1	27.71	19.74
Sep	35	1	28.04	36.26
Oct	45	0	25.62	64.00
Nov	53	0	18.52	94.79
Dec	57	0	16.53	112.42

### 3.3 Effect of earth reflection coefficient on the maximum radiation

In this part, the effect of earth reflective coefficient on the optimum slope and azimuth angles and the maximum solar radiation are investigated. The results show that the maximum total radiation on the collector increases due to the increase in earth reflection coefficient.

### **Conclusions**

- 1. The maximum and minimum optimum slope angles were obtained as  $-1.42^{\circ}$  and  $60.36^{\circ}$ , respectively using isotropic Liu mode in Mazandaran.
- 2. Yearly optimum slope angle was obtained to 30.12 in Mazandaran. This angle is nearby the latitude angle of Mazandaran.
- 3. The results showed that total radiation during a year with optimum slope angle comparing to total radiation during a year on flat surface increased 25.4%.





- 4. The results showed that total radiation during a year with optimum slope and azimuth angles comparing to total radiation during a year on flat surface increased 31.28% due to KT model. This was 57.76% for Hay model.
- 5. The results showed that the maximum radiation reached to the collector increases with the increasing of the earth reflection coefficient.
- 6. The results showed that the collector must be installed in a region of Mazandaran with the high earth reflection coefficient.

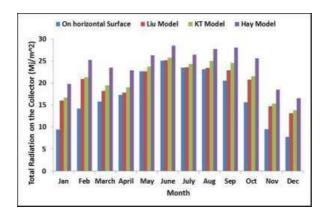


Figure6- Comparison of maximum radiation on the collector in different cases

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### بهینه سازی عملکرد کلکتور خورشیدی با تعیین زوایای بهینه شیب، سمت سطح و بررسی اثر ضریب انعکاس زمین بر عملکرد آنها در استان مازندران

### چکیده

عملکرد یک کلکتور خورشیدی با قرار گیری آن تحت زوایای بهینه شیب، سمت سطح و ضریب انعکاس زمین بهبود پیدا می کند. در مقاله حاضر زوایای بهینه شیب و سمت سطح ماهیانه یک کلکتور خورشیدی با استفاده از مدل های KT ، Hay می کند. در استان مازندران بدست آورده شد. نتایج نشان داد با قرارگیری کلکتور تحت زوایای بهینه شیب و سمت سطح با توجه به مدل KT انرژی جذب شده توسط کلکتور ۲۸/۲۸٪ و با توجه به مدل ۷۷/۷۲ افزایش پیدا می کند. همچنین نتایج نشان داد که با افزایش ضریب انعکاس زمین مقدار انرژی جذب شده توسط کلکتور با توجه به مدل Hay افزایش پیدا می کند. ضمنا مکان مناسب برای نصب کلکتور جایی است که ضریب انعکاس زمین بیشتر باشد.

واژههای کلیدی: انرژی خورشیدی، کلکتور خورشیدی، زاویه شیب بهینه، زاویه سمت بهینه، ضریب انعکاس زمین